

# Radiative Transfer in Entry Flows

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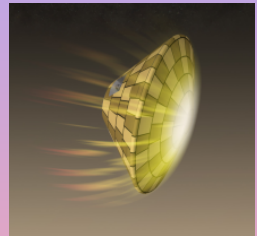
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15 June 2013



# Introduction

- Radiative heating can be a major design driver for large or high-speed ( $\geq 10\text{km/s}$ ) entry vehicles
- For lower entry speeds, radiative heating may mandate additional thermal protections (e.g. base heating for Martian entries)
- Convective heating mostly depends on ground species, radiative heating depends on excited species. Larger uncertainties
- Accounting for an additional spectral dimension with grids with over  $10^6$  points. Very computationally intensive CFRD simulations.

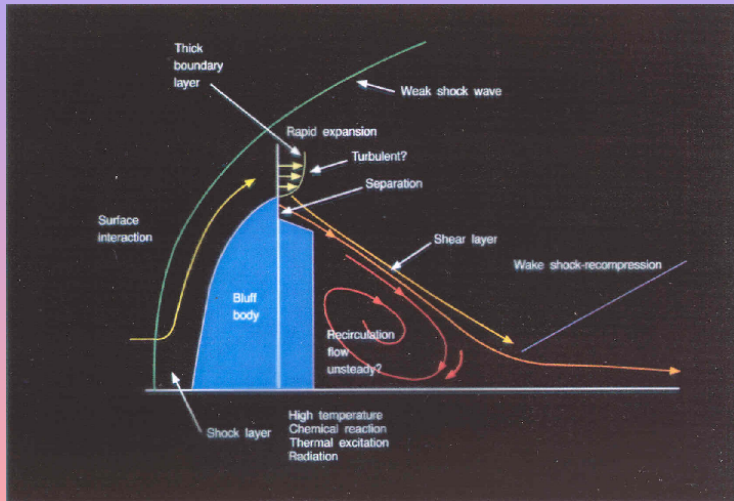


# Main Drivers

- Do we need to take radiation into account?
- Optically thin or thick?
- Coupling between flow and radiation?
- Equilibrium or non-equilibrium radiation?
- Selection of an appropriate radiative database
- Other issues (precursor, photo-chemistry, photo-ionisation processes)



# Aerothermodynamics of Entry Vehicles



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## Fluid

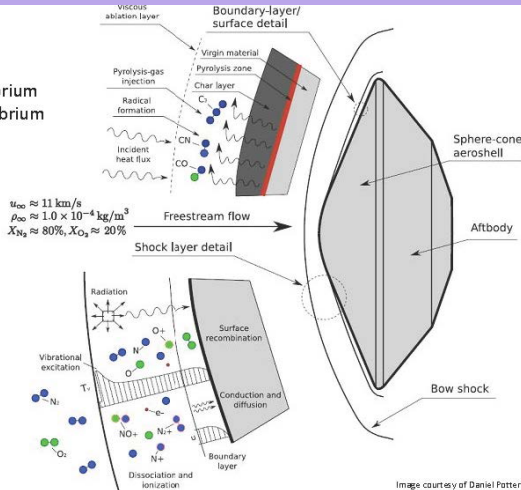
- Thermal non-equilibrium
- Chemical non-equilibrium
- Ionisation

## Heat shield

- Pyrolysis
- Charring
- Mechanical erosion

## Surface

- Recombination
- Re-radiation
- Surface reactions



# Engineering Correlations for Radiation

- Engineering Correlations useful when exploring different EDL scenarios
- Exploratory parameters: Spacecraft sizing, atmospheric profiles and composition, entry path and angle, aerodynamic coefficients, etc...
- Correlations then complemented with detailed CFRD calculations for pre-selected Flight paths, at specific representative entry points ( $\sim 5$ )



# Blackbody radiation limit

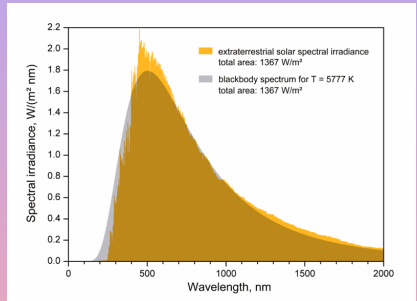
spectral emissivity of a gas has a theoretical limit given by Planck's law

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2} \left[ \exp\left(\frac{h\nu}{k_B T}\right) - 1 \right]^{-1}$$

The spectral integration yields the Stefan–Boltzmann Law:

$$\int_{\nu} B_{\nu}(T) = \sigma T^4$$

Engineering correlations are based on this upper theoretical limit for the emitted radiation



The Sun's emissivity is close to a blackbody

# Stagnation Streamline flux dependence on cone radius

- Convective fluxes**

Sutton and Graves, Fay and Riddel correlations:

$$q_{conv}^w = f \left[ \sqrt{\left( \frac{du}{dx} \right)_s} \right]$$

from Newtonian theory:  $\left[ \left( \frac{du}{dx} \right)_s \right] = \frac{1}{R} \sqrt{\frac{2p - p_\infty}{\rho}}$

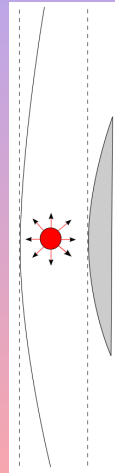
$$q_{conv}^w \propto \frac{1}{\sqrt{R}}$$

- Radiative Fluxes**

Optically Thin shock layer:  $q_{rad}^w = \left( \frac{E\delta}{2} \right)^a$  with  $\delta \simeq \frac{R}{\rho_s/\rho_\infty}$

$$q_{rad}^w \propto R$$

Requirements for minimizing convective and radiative heat fluxes are mutually exclusive



<sup>a</sup>E: radiated power, half goes upstream, half to the wall.  $\delta$  is the shock standoff. s: shock



# Stagnation Streamline flux dependence on entry speed

## ● Convective fluxes

$$q_{conv}^w \approx 1/2 \rho_{\infty} u_{\infty}^3 St^a$$

$$q_{conv}^w \propto u_{\infty}^3$$

## ● Radiative Fluxes

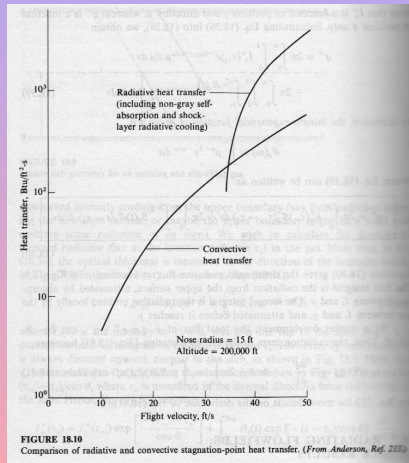
$$T_s = \frac{u_{\infty}^2}{2C_{ps}};$$

$$q_{rad}^w = \sigma T_s^4 = \frac{u_{\infty}^8}{(2C_{ps})^4}$$

$$q_{rad}^w \propto u_{\infty}^8$$

Radiation dominates above  
10km/s

<sup>a</sup>St: Stanton number



# Boltzmann and Goulard Numbers

Boltzmann number  $B_o^{-1}$ , Radiation/Convection ratio:

$$B_o^{-1} = \frac{q_{rad}}{q_{conv}} = \frac{\sigma T^4}{\rho u C_p T}$$

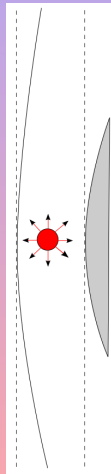
Infinite slab approximation<sup>a</sup>. We also have  $C_p T \simeq u_\infty/2$ .

Goulard Number  $\Gamma$ , Radiative cooling parameter

$$\Gamma = \frac{2q_{rad}^w}{q_{conv}^w} = \frac{2q_{rad}^w}{1/2\rho_\infty u_\infty^3}$$

$\sigma$ : Stefan-Boltzmann constant

<sup>a</sup> half the radiated power goes upstream, half to the wall

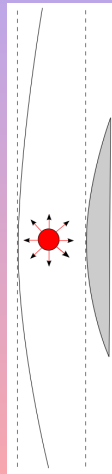


# Optically Thin and Optically Thick Limiting Cases

- **Optically Thin:**  $\Gamma_n = \frac{4\sigma T_s^4 \alpha \delta}{\rho_\infty u_\infty h_s}$
- **Optically Thick<sup>a</sup>:**  $\Gamma_k = \frac{16\sigma T_s^4 \alpha}{3\rho_\infty u_\infty h_s}$
- $\Gamma < 10^{-3}$ : radiation is not important
- $10^{-3} < \Gamma < 10^{-2}$ : radiation is important but coupling is not necessary
- $\Gamma > 10^{-2}$ : coupling of the radiation to the flowfield is necessary

The absorption coefficient is related to the optical transmissivity such that  $T = \exp(-\alpha\delta)$ . Transmissivity goes from 100% (optically thin) to 0% (optically thick).

<sup>a</sup> known as the Rosseland limit



# Radiative Transfer Regimes

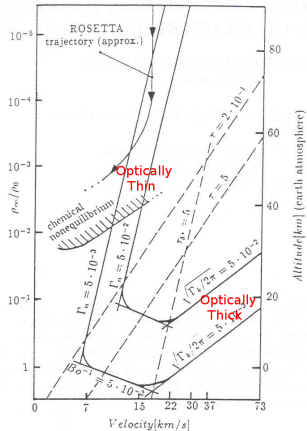
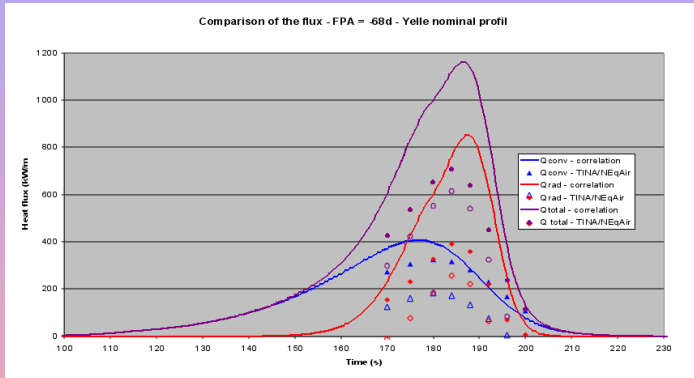


Figure 2: Radiative transfer regimes for a blunt body of 1 ft nose radius, taken from [2]

# Sample Correlated Heat Fluxes for the Huygens Entry

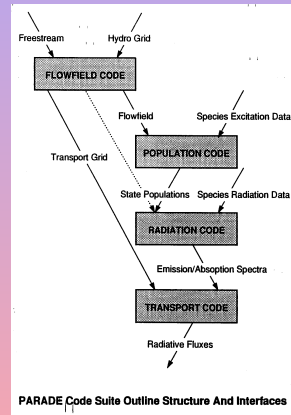


Correlated and simulated convective heat fluxes for the Huygens entry

Engineering correlations allow a first sampling of the spacecraft shape and the selection of an appropriate descent trajectory. Then CFRD simulations can be carried out for specific trajectory points

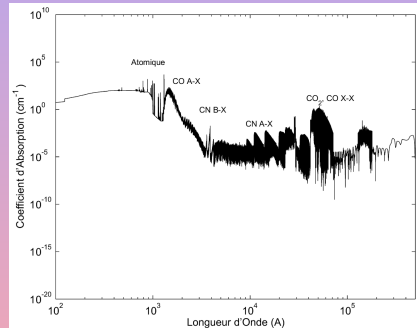
# Computational Fluid Radiative Dynamics Simulations

- Development of radiative databases providing the spectral-dependent emission  $\epsilon_\nu$  and absorption  $\alpha(\nu)$  coefficients for any arbitrary state of the plasma
- Accounting for bound, bound-free (photodissociation, photoionization, photodetachment) and free-free (Bremstrahlung) transitions
- bound-bound transitions include atomic, diatomic and polyatomic transitions
- Radiative transfer equation is then solved over the computational domain



# Development of Spectral Databases

- atomic and polyatomic transitions typically simulated from external radiative databases (NIST, TopBase, HITRAN)
- diatomic transitions databases are explicitly obtained using appropriate quantum-mechanics models
- continuum radiation calculated from published absorption cross-sections (e.g. TopBase for atomic photoionization), or from semi-empirical to exact cross-sections for molecular continuum cross-sections



Absorption Coefficient for a 97%CO<sub>2</sub>–3%N<sub>2</sub> Plasma in  
Thermochemical Equilibrium at 5000K and 1bar

esa



# Line-by-Line Models

- “Collection” of lines which correspond to the overall quantum-allowed radiative transitions between the internal levels of an atom/molecule.
- Selection rules define the quantum-allowed transitions, which must respect angular and spin momentum conservation for atoms and molecules.
- Three key parameters:
  - 1) Line position considering Planck's Law:  $\nu = E_u - E_l/hc$ ;
  - 2) Line intensity:  $I_\nu = N_u A_{ul} \Delta E_{ul}$ ;
  - 3) Line profile:  $F(\nu - \nu_0)$ , Voigt (sum of a Lorentz & Gaussian profile).
- Radiative spectra obtained through the superposition of these overall lines



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# Key Parameters

The fundamental equation for intensity

$$I_{\nu} = N_u A_{ul} \Delta E_{ul} F(\nu - \nu_0)$$

can be decoupled in the following variables :

- $N_u$  : Level populations :

Either a Boltzmann equilibrium distribution, either through Collisional-Radiative nonequilibrium models

- $A_{ul}$  : Transition probabilities (Einstein Coefficients):

From "ab-initio" or measured transition moments

- $\Delta E_{ul}$  : Level energies:

Fitting and extrapolation of experimentally determined data

- $F(\nu - \nu_0)$  : Line profile:

Voigt profiles accounting for Doppler and Collisional broadening (depends on the local conditions of the plasma)



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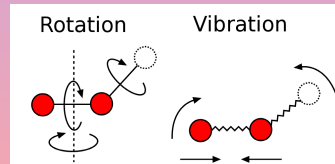
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**Voigt profiles** accounting for Doppler and Collisional broadening (depends on the local conditions of the plasma)

# Rotational-Vibrational Level Energies of Diatomic Molecules

$$\begin{aligned}
 E_{e,v,J} &= T_e + G(v) + B_v(J) \\
 &= T_e + \omega_e (v + 1/2) - \omega_e x_e (v + 1/2)^2 + \omega_e y_e (v + 1/2)^3 + \dots \\
 &\quad + \left[ B_e - \alpha_e (v + 1/2) + \gamma_e (v + 1/2)^2 + \dots \right] [J(J + 1)] \\
 &\quad - [D_e + \beta_e (v + 1/2) + \dots] [J(J + 1)]^2 + \dots
 \end{aligned}$$

In the compact Dunham matrix form  
(strictly valid only for  $^1\Sigma$  states):

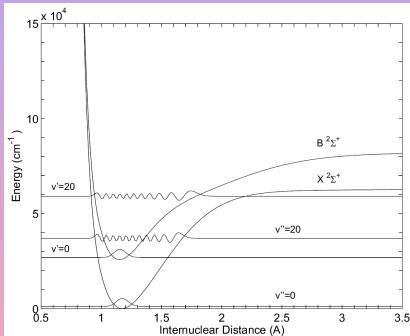
$$E_{e,v,J} = \sum_{i,j} Y_{ij} (v + 1/2)^i [F(J)]^j,$$



Molecular vibrational and rotational levels

# Potential Reconstruction Methods

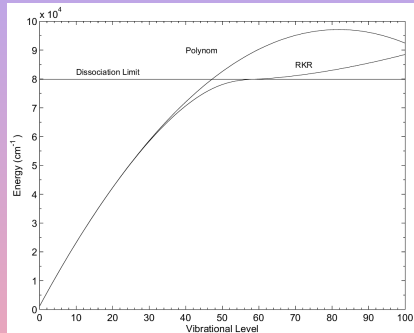
- Potential reconstruction methods allow determining the radial wavefunction for the molecular vibrational levels.
- They also incidentally allow more accurate representations of near-dissociative levels, where polynomial expansions are no longer valid.



Sample RKR calculation for the CN Violet system

# One Sample Application

- Calculation of the vibrational levels for the  $N_2(X^1\Sigma)$  ground state.
- Traditional polynomial expansions predict  $v_{max}=46$ .
- A more rigorous RKR potential reconstruction method yields  $v_{max}=61$ .



$N_2(X^1\Sigma)$  Level energies calculated using a second order polynomial expansion, and the RKR method



# Fine-structure Effects (1/4)

- Fine-structure effects like spin splitting and  $\Lambda$ -Doubling for homonuclear molecules, split one single line into multiplet lines with energies very close one to another.
- Nuclear momentum is named as  $\Sigma$ ,  $\Pi$ ,  $\Delta$ , etc.,. for  $\Lambda = 0, 1, 2$
- The level energies expressions for spin splitted lines are summarized next

# Fine-structure Effects (2/4)

- Singlet States with  $\Lambda \neq 0$

$$F_v(J) = B_v(J(J+1)) + (A_v - B_v)\Lambda^2$$

- Doublet States

$$F_{3/2}(J \geq 1) = B_v \begin{bmatrix} (J + \frac{1}{2})^2 - \Lambda^2 \\ -\frac{1}{2} \left( 4(J + \frac{1}{2})^2 + Y_v(Y_v - 4)\Lambda^2 \right)^{\frac{1}{2}} \end{bmatrix} - D_v J^4$$

$$F_{1/2}(J \geq 0) = B_v \begin{bmatrix} (J + \frac{1}{2})^2 - \Lambda^2 \\ +\frac{1}{2} \left( 4(J + \frac{1}{2})^2 + Y_v(Y_v - 4)\Lambda^2 \right)^{\frac{1}{2}} \end{bmatrix} - D_v (J+1)^4$$

# Fine-structure Effects (3/4)

## • $^3\Sigma$ Triplet States

$$\begin{aligned}
 {}^3\Sigma_2(J \geq 2) &= B_v(J(J+1)) - D_v(J(J+1))^2 - \left( \lambda_v - B_v + \frac{1}{2}\gamma_v \right) \\
 &\quad - \left[ \left( \lambda_v - B_v + \frac{1}{2}\gamma_v \right)^2 + 4J(J+1) \left( B_v - \frac{1}{2}\gamma_v \right)^2 \right]^{\frac{1}{2}} \\
 {}^3\Sigma_1(J \geq 1) &= B_v(J(J+1)) - D_v(J(J+1))^2 \\
 {}^3\Sigma_0(J \geq 0) &= B_v(J(J+1)) - D_v(J(J+1))^2 - \left( \lambda_v - B_v + \frac{1}{2}\gamma_v \right) \\
 &\quad + \left[ \left( \lambda_v - B_v + \frac{1}{2}\gamma_v \right)^2 + 4J(J+1) \left( B_v - \frac{1}{2}\gamma_v \right)^2 \right]^{\frac{1}{2}}
 \end{aligned}$$

## Fine-structure Effects (4/4)

### • $^3\Pi$ Triplet States

$$^3\Pi_2(J \geq 2) = B_v \left[ \frac{J(J+1) - \sqrt{y_1 + 4J(J+1)}}{-\frac{2}{3} \frac{y_2 - 2J(J+1)}{y_1 + 4J(J+1)}} \right] - D_v \left( J - \frac{1}{2} \right)^4$$

$$^3\Pi_1(J \geq 1) = B_v \left[ J(J+1) + \frac{4}{3} \frac{y_2 - 2J(J+1)}{y_1 + 4J(J+1)} \right] - D_v \left( J + \frac{1}{2} \right)^4$$

$$^3\Pi_0(J \geq 0) = B_v \left[ \frac{J(J+1) + \sqrt{y_1 + 4J(J+1)}}{-\frac{2}{3} \frac{y_2 - 2J(J+1)}{y_1 + 4J(J+1)}} \right] - D_v \left( J + \frac{3}{2} \right)^4$$

with

$$y_1 = Y_v(Y_v - 4) + \frac{4}{3} \quad y_2 = Y_v(Y_v - 1) - \frac{4}{9} \quad Y_v = \frac{A_v}{B_v}$$

# Selection Rules for Atoms

	Transitions E1	Transitions M1	Transitions E2
<b>Tous types de couplages</b>			
1	$\Delta J = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ )	$\Delta J = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ )	$\Delta J = 0, \pm 1, \pm 2$ (sauf $0 \leftrightarrow 0$ , $\frac{1}{2} \leftrightarrow \frac{1}{2}, 1 \leftrightarrow 1$ )
2	$\Delta M = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ lorsque $J = 0$ )	$\Delta M = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ ) lorsque $J = 0$ )	$\Delta M = 0, \pm 1, \pm 2$
3	changement de parité	parité identique	parité identique
4 <sup>1</sup>	transition d'un électron avec $\Delta l = \pm 1$ , $\Delta n$ arbitraire	pas de changement dans la configuration électronique, soit pour tous les électrons: $\Delta l = 0, \Delta n = 0$	pas de changement dans la configuration électronique, ou bien pour un électron: $\Delta l = 0, \pm 2$ , $\Delta n$ arbitraire
<b>Couplage L-S</b>			
5	$\Delta S = 0$	$\Delta S = 0$	$\Delta S = 0$
6	$\Delta L = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ )	$\Delta L = 0, \Delta J = \pm 1$	$\Delta L = 0, \pm 1, \pm 2$ (sauf $0 \leftrightarrow 0$ , $0 \leftrightarrow 1$ )

<sup>1</sup>Lorsque l'interaction de configuration est négligeable.

# Selection Rules for Molecules

transition entre niveaux de rotation	$\Delta J = 0, \pm 1$ (sauf $0 \leftrightarrow 0$ )
parité des niveaux de rotation	$+$ $\leftrightarrow$ $-$ permis $+$ $\leftrightarrow$ $+$ , $-$ $\leftrightarrow$ $-$ interdits
branches de rotation $Q(\Delta J = 0)$	$e \leftrightarrow f$ permis $e \leftrightarrow e$ , $f \leftrightarrow f$ interdits
branches de rotation $P, R(\Delta J = \pm 1)$	$e \leftrightarrow e$ , $f \leftrightarrow f$ permis $e \leftrightarrow f$ interdits
molécules homonucleaires	$s \leftrightarrow s$ , $a \leftrightarrow a$ permis $s \leftrightarrow a$ interdits
noyaux de même charge	$g \leftrightarrow u$ permis $g \leftrightarrow g$ , $u \leftrightarrow u$ interdits

# Transition Probabilities (1/2)

Transition probability  $A_{ul}$  decomposed into vibronic and rotational product

$$A_{ul} = A_{e''v''}^{e'v'} \cdot A_{\Lambda''J''}^{\Lambda'J'}$$

with

$$A_{e''v''}^{e'v'} = \frac{64\pi^4 \bar{\nu}^3}{3hc^3} \frac{(2 - \delta_{0,\Lambda'+\Lambda''})}{(2 - \delta_{0,\Lambda'})} \left( R_e^{v'v''} \right)^2$$

and

$$\left( R_e^{v'v''} \right)^2 \cong \frac{\sum \left( R_e^{v'v''} \right)^2}{(2 - \delta_{0,\Lambda'+\Lambda''})(2S + 1)}$$

The vibronic transition moment being obtained from the integration of the electronic transition moment  $R_e(r)$  and the upper and lower states wavefunctions

$$\left( R_e^{v'v''} \right)^2 = \left( \int \psi_{v'}(r) R_e(r) \psi_{v''}(r) dr \right)^2$$

## Transition Probabilities (2/2)

The rotational transition probability is given by the Hönl-London factors

$$A_{\Lambda''J''}^{\Lambda'J'} = \frac{S_{\Lambda''J''}^{\Lambda'J'}}{2J' + 1}$$

which depend on the type of rotation-electronic motion coupling (Hund Case), and the type of electronic transition ( ${}^n\Lambda \leftrightarrow {}^n\Lambda$ ).

We use the normalization rule

$$\sum_{J''} S_{\Lambda''J''}^{\Lambda'J'}(J') = (2J' + 1)$$

and we may then write

$$A_{ul} = \frac{64\pi^4 \bar{\nu}^3}{3hc^3} \frac{\sum \left( R_e^{v'v''} \right)^2}{(2 - \delta_{0,\Lambda'}) (2S + 1)} \frac{S_{\Lambda''J''}^{\Lambda'J'}}{2J' + 1}$$

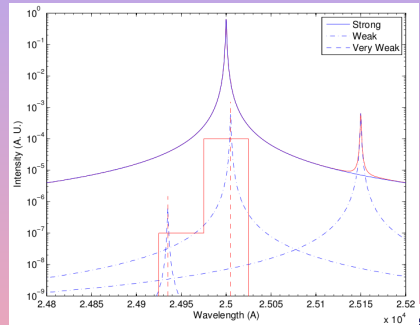


# Voigt Lineshape Calculation Method

- Line broadening mechanisms from Doppler or collision effects (Lorentz lineshape).
- Objective: To calculate the most accurate Voigt lineshape (convolution of Doppler and Lorentz lineshapes) utilizing a minimum number of points.
- Input parameters: line position, emission/absorption coefficient, Doppler/Lorentz FWHM.
- Starting point: Analytical expression of Whiting, updated by Olivero (JQSRT, Vol. 17, 1977, pp. 233).
- Definition of low-resolution lines (5 to 11 grid points) and high-resolution lines (typically more than 20 points).

# Definition of Strong and Weak Lines

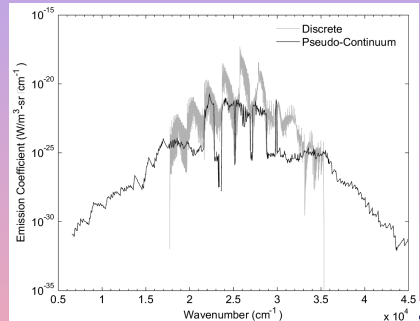
- Definition of strong, weak, and very weak lines.
- In the “pseudo-continuum” approach, only strong lines and uncovered weak lines are calculated explicitly. Other lines are added as a continuum.



Sample strong, weak, and very weak lines (with thresholds  $10^{-5}$  and  $10^{-6}$ )

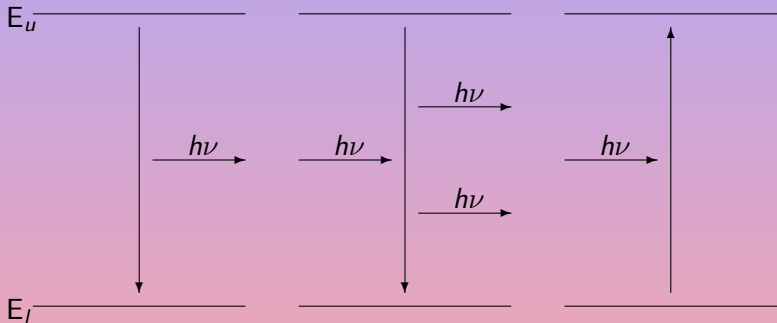
# Sample Calculation for the CN Violet System

- Here, only the strong lines for diagonal transitions are explicitly calculated.
- The “pseudo-continuum” extends below.
- Significant savings in calculation times and spectral grid sizes can be achieved (Lino da Silva, JQSRT 2007)



Calculation of the molecular CN Violet System using the hybrid line-by-line/pseudo-continuum approach.

# Elementary Radiative Processes



spontaneous emission   induced emission

absorption

$$\frac{\partial N_u}{\partial t} = -N_u A_{ul}$$

$$\frac{\partial N_u}{\partial t} = -N_u B_{ul} u_\nu$$

$$\frac{\partial N_l}{\partial t} = -N_u B_{ul} u_\nu$$

# Derivation of the Radiative Transfer Equation (RTE) (1/2)

Summing the three processes we have

$$\frac{\partial N_u}{\partial t} = -N_u A_{ul} + [N_u B_{ul} - N_u B_{ul}] u_\nu$$

In Radiative equilibrium the net variation is 0, and radiation is given by the **Planck Blackbody Law**

$$u_\nu^o(T) = \frac{8\pi h\nu^3}{c^3} \left[ \exp\left(-\frac{h\nu}{k_B T}\right) - 1 \right]^{-1}$$

we have

$$g_l B_{lu} = g_u B_{ul}$$

$$\frac{A_{ul}}{B_{ul}} = \frac{8\pi h\nu^3}{c^3}$$

# Derivation of the Radiative Transfer Equation (RTE) (2/2)

Integrating the expressions of the previous slide in volume from we obtain the **classical RTE equation**:

$$\cos \theta \frac{dL_\nu}{dz} = \varepsilon_\nu - \alpha_\nu L_\nu$$

with

$$\varepsilon_\nu = \frac{n_u A_{ul} h\nu}{4\pi}$$

$$\alpha(\nu) = \left( n_u - \frac{g_u}{g_l} n_l \right) B_{ul} \frac{h\nu}{c}$$

which becomes the **Beer-Lambert Law** in 1D coordinates:

$$\frac{dl_\nu}{dl} = \varepsilon - \alpha(\nu) l_\nu$$

and after integration:

$$l_\nu = \frac{\varepsilon}{\alpha(\nu)} [1 - \exp(-\alpha(\nu)l)]$$



# Detailed Balancing Principle for Radiation

- **Bound-Bound** Transitions

$$\frac{\varepsilon_\nu}{\alpha_\nu} = \frac{2h\nu^3}{c^2} \left( \frac{g_u n_l}{g_l n_e} - 1 \right)^{-1}$$

- **Bound-Free** Transitions

$$\frac{\sigma_{bf}(\nu)}{\sigma_{fb}(\nu)} = \frac{1}{2} \left( \frac{m_e v_e c}{h\nu} \right)^2 \frac{g_+ g_e}{g_n}$$

- **Free-Free** Transitions

$$\frac{\varepsilon_\nu}{\alpha_\nu} = \frac{2h\nu^3}{c^2} \left[ \exp \left( \frac{h\nu}{k_B T_e} \right) - 1 \right]^{-1}$$

# Detailed Balancing Principle for Radiation

- **Bound-Bound** Transitions

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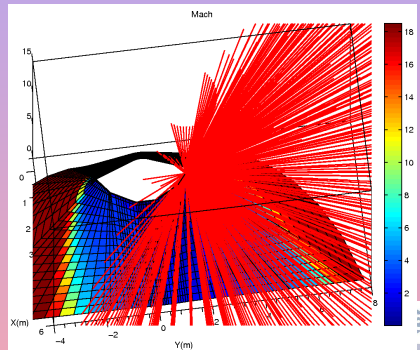
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# Ray-Tracing Methods for Space-Related Heat Transfer Analysis

- Ray-Tracing: An exact method for determining the quantities of photons which irradiate a surface.

$$q_\nu = \int_{-\pi/2}^{\pi/2} \int_0^\pi I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi$$

$$I_\nu = \frac{\epsilon}{\alpha'} [1 - \exp(-\alpha l(\theta))]$$



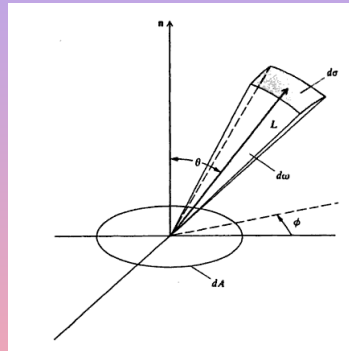
- Ray-tracing considers full 3D geometries, with the possibility of simpler 2D/1D slab geometries.

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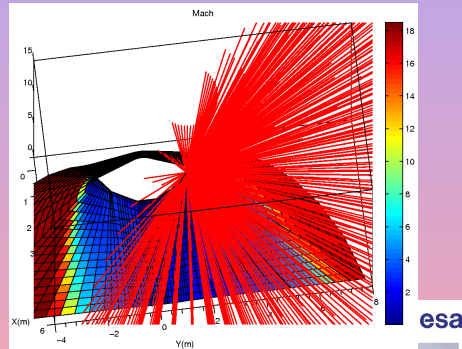
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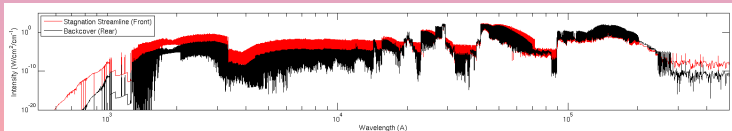
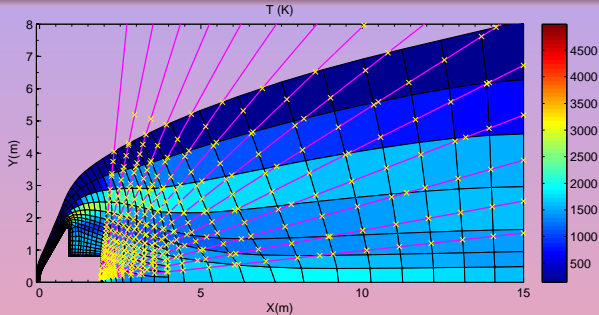
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# Ray-Tracing Methods for Space-Related Heat Transfer Analysis



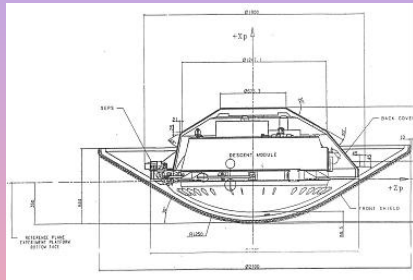
Complex flow geometries with **millions** of molecular spectral lines

# Case Studies

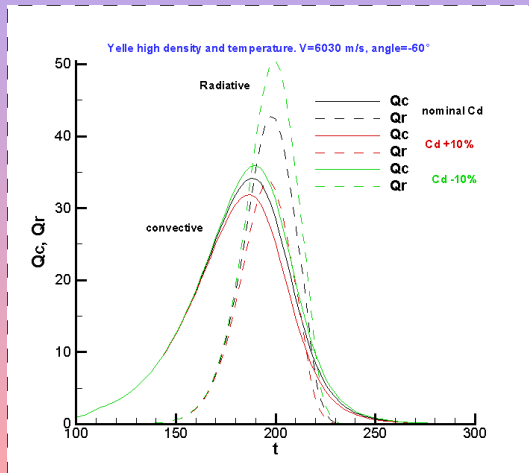


# Huygens Titan Entry (2005)

- 60 half angle blunted cone
- Base diameter: 2.7 m
- Nose radius: 1.25 m
- Entry module mass: 318.62 kg
- Aerodynamic databases developed by EADS between 1991 and 1995.

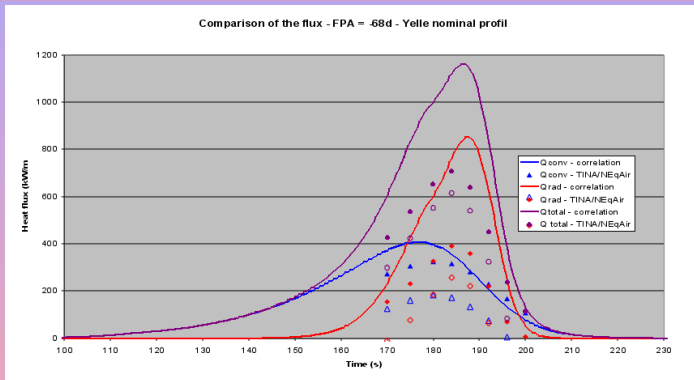


# Huygens Heat Fluxes: Dispersions associated to Cd uncertainties



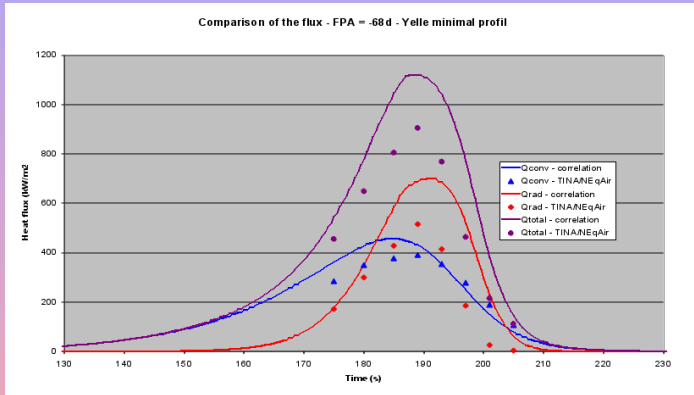


# Huygens Heat Fluxes: Nominal Yelle Atmospheric Profile



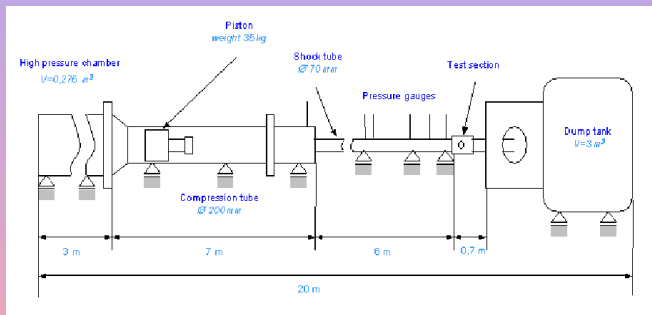
- Titan Composition: 95%  $N_2$ –5%  $CH_4$
- Important Radiator species CN produced in the shock layer. correlations do not work anymore.

# Huygens Heat Fluxes: Nominal Yelle Atmospheric Profile



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# Validation of Aerothermodynamic Databases: Shock-Tubes Experiments

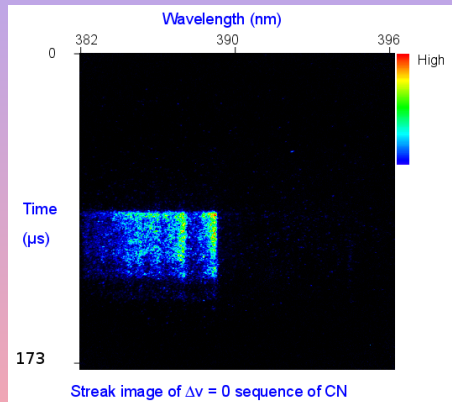


- TCM2 Shock-Tube at Marseilles, France, was utilized for validating the CFD models used for Huygen's Entry

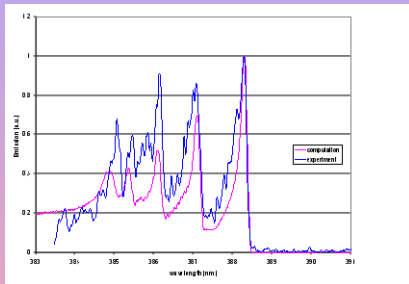
# Acquisition Results: CN spectral time-resolved picture

## Shot conditions :

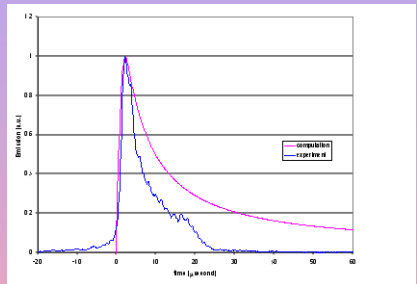
- Shock Tube  
Classical mixture  
 $p_{shock} = 200$  Pa
- Shock velocity  
 $v_{shock} = 5680$  m/s



## 40 Pa test rebuilding, Boltzmann



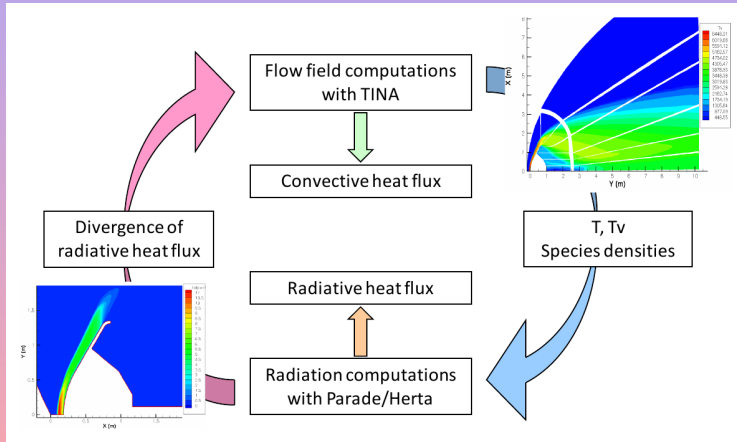
Non-equilibrium zone, 1s integration



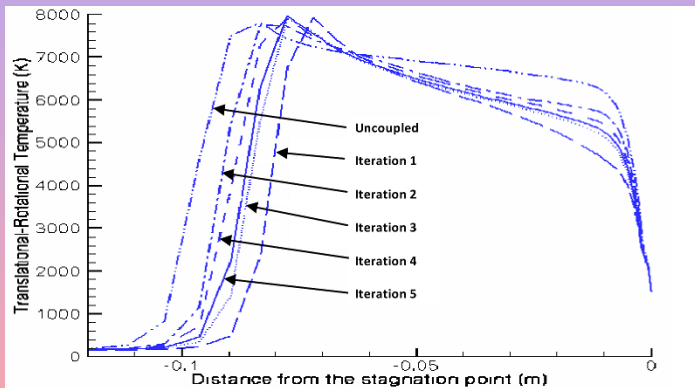
Temporal evolution

Validation of Aerothermal databases against simpler (1D, time-dependent) representative flow conditions

# Coupled CFD-Radiative Simulations of Huygens Entry



# Stagnation Streamline Temperatures Variation During CFD-Radiation Iterations



# Spectral Convergence

**Table 1: Influence of number of points for radiative heat flux**

		Without Absorption		With Absorption	
Nb points	Resolution (pt/Å)	Q rad kW/m <sup>2</sup>	Variation %	Q rad kW/m <sup>2</sup>	Variation %
5 000	0.28	2091.33	+0.05	1773.38	+30.58
50 000	2.78	2091.00	+0.03	1603.38	+18.06
100 000	5.56	2090.96	+0.03	1535.54	+13.07
200 000	11.1	2090.59	+0.01	1488.07	+9.57
1 800 000	100	2090.28	-	1358.08	-



# Influence of Radiation and Radiation Coupling

Radiative heat flux		Without coupling	With coupling
Without Absorption	kW / m <sup>2</sup>	813.82	516.82
With Absorption	kW / m <sup>2</sup>	558.41	401.87

Variation		Without coupling	With coupling
Without Absorption	%	-	-36%
With Absorption	%	-31%	-51%

# Phoebus High-Speed Entry Demonstrator

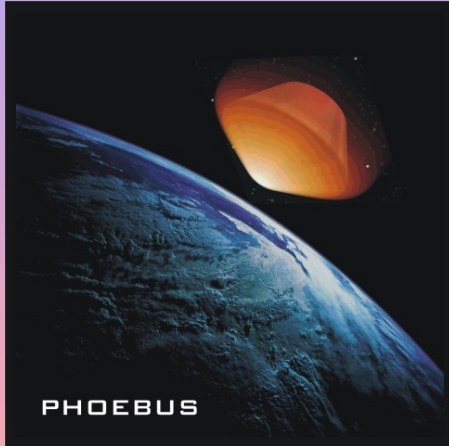
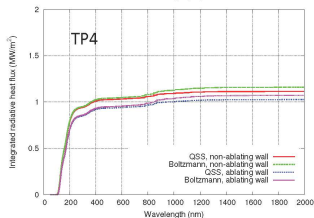
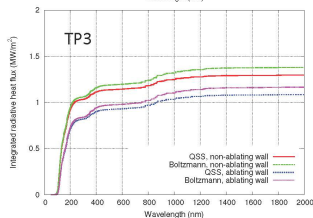
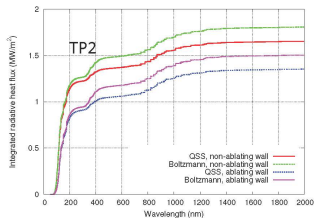
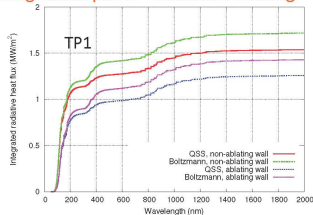


Table 9.2: *Phoebus* simulation trajectory points and specifics of the corresponding freestream

Name	Time	Altitude	$\rho_{\infty}$ (kg/m <sup>3</sup> )	$T_{\infty}$ (K)	$p_{\infty}$ (Pa)	$u_{\infty}$ (m/s)	Mach
TP1	18.4	64981	1.64e-4	233.3	10.96	10916	35.65
TP2	21.8	55449	4.23e-4	254.0	30.96	10044	32.04
TP3	24.1	49464	1.10e-3	270.6	85.28	9518	28.86
TP4	26.0	45004	2.00e-3	264.2	149.10	8456	25.95

# Coupled Radiative Calculations with Ablation and Boltzmann Nonequilibrium Effects

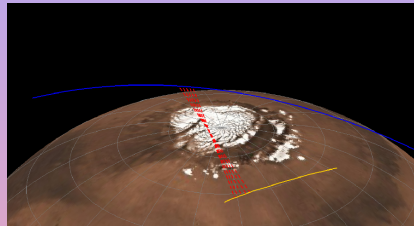
Stagnation point – Radiative heating



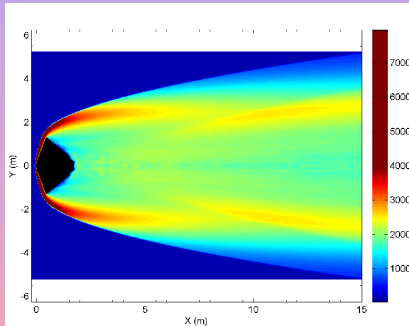
Courtesy O. Joshi (EPFL)

# Mars EXPRESS Observation of the PHOENIX Entry

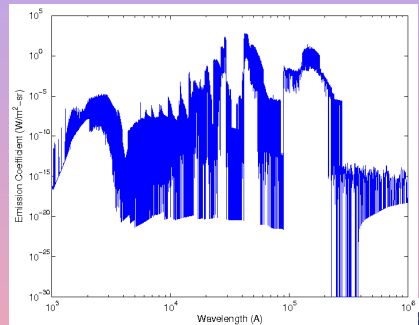
- ESA Mars EXPRESS Orbiter provided support for PHOENIX 2008 Mars Entry
- Mars EXPRESS Tracked PHOENIX Entry with his onboard instrumentation (First Attempt at tracking an Entry from an Onboard Satellite).
- CFRD simulations prior to entry predicted the IR trail to be the most emissive.
- SPICAM VUV Camera and HRSC Visible Camera tracked the Entry. IR Fourier Spectrometer could not be turned on due to power budget constraints.



# CFRD Simulations of PHOENIX Entry

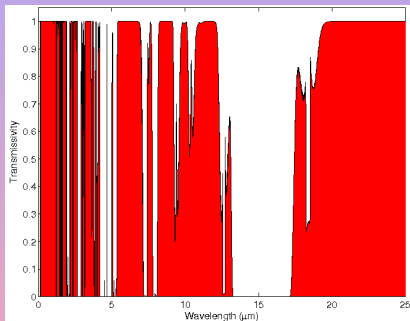


PHOENIX Temperature Profiles,  $t=203s$

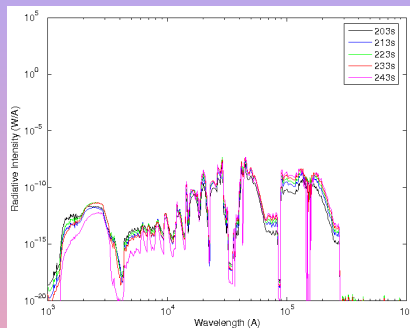


Sample spectrum of the PHOENIX Plume

# Radiative Transfer Towards Mars EXPRESS



Optical path Transmissivity



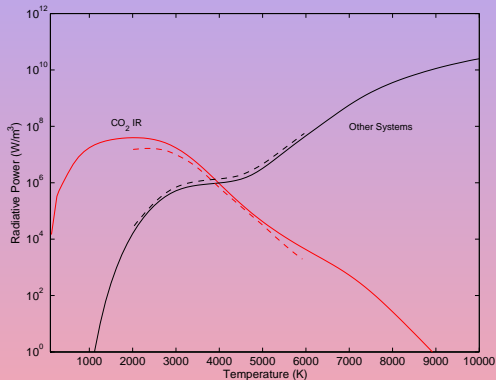
Radiation collected by one pixel of the Mars EXPRESS instrumentation

Radiation in the VUV-Visible region was not detected, IR radiation was predicted to be detected if the instrument had been switched on

# Spectral Database for CO<sub>2</sub>-N<sub>2</sub> Mixtures

- CO<sub>2</sub> Infrared
- CO Infrared
- CO 4<sup>+</sup>
- CO 3<sup>+</sup>
- CO Angstrom
- CO Triplet
- CN Violet
- CN Red
- C<sub>2</sub> Swan
- C<sub>2</sub> Phillips
- C<sub>2</sub> Mulliken
- C<sub>2</sub> Deslandres-D'Azambuja
- C<sub>2</sub> Ballik-Ramsay
- O<sub>2</sub> Sch.-Runge
- O<sub>2</sub> Sch.-Runge Cont.
- N<sub>2</sub> 1<sup>+</sup>
- N<sub>2</sub> 2<sup>+</sup>
- NO Gamma
- NO Beta
- NO Delta
- NO Epsilon
- NO Beta
- C Atomic
- C Photoionization
- N Atomic
- N Photoionization
- O Atomic
- O Photoionization
- C<sup>-</sup> Photodetachment
- N<sup>-</sup> Photodetachment
- O<sup>-</sup> Photodetachment
- CO<sub>2</sub> Photoionization
- N<sub>2</sub> Photoionization
- O<sub>2</sub> Photoionization
- CN Photoionization
- CO Photoionization
- NO Photoionization

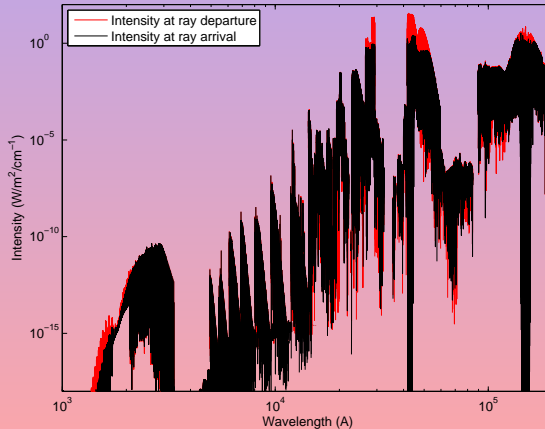
# Radiative Power for CO<sub>2</sub>-N<sub>2</sub> Mixtures: Comparison With Other Spectral Databases



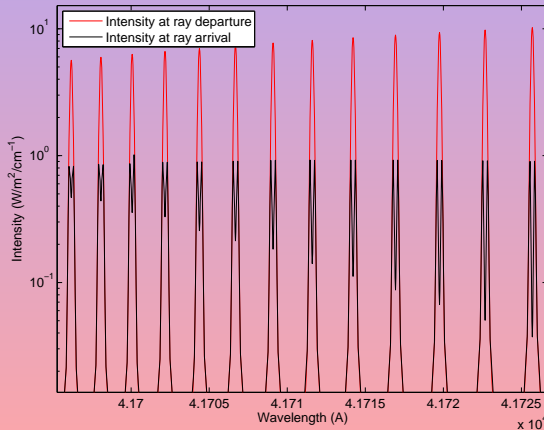
Comparison of the overall temperature dependent radiative power of CO<sub>2</sub> IR radiation (red) and for the other radiative systems (black) for an atmospheric pressure, Martian-type CO<sub>2</sub>-N<sub>2</sub> plasma. Comparison is carried for the SPARTAN code database (full lines) and the EM2C database (dotted lines).



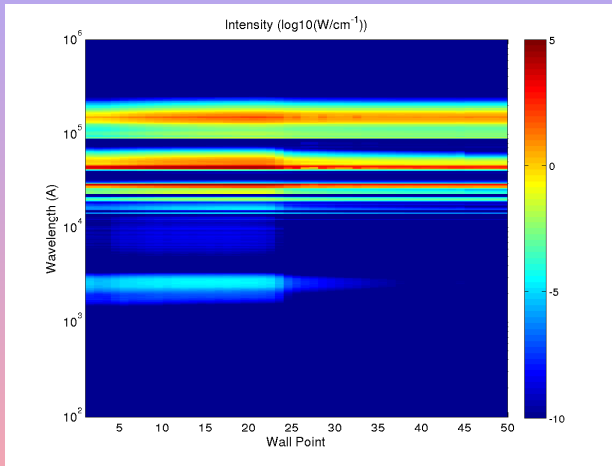
# Example of a Ray Departing from the Afterbody Plume; PH (1/2)



# Example of a Ray Departing from the Afterbody Plume; PH (2/2)

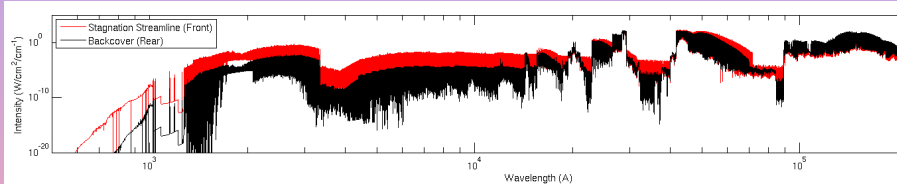


# Spectral Distribution of the Wall Fluxes; PH



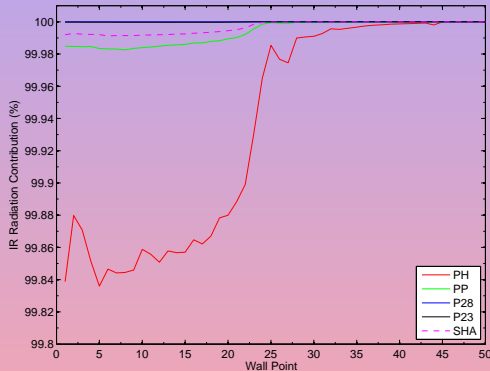
CO<sub>2</sub> IR radiation is dominant, with traces of CO 4<sup>+</sup>, CO 3<sup>+</sup>, and CO Angstrom in the forebody

# Comparison of Stagnation Streamline and Backcover Shoulder Spectra; PH



- VUV-Visible radiation higher for the stagnation streamline points
- For both points,  $\text{CO}_2$  radiation spectral intensity is several orders of magnitude above Visible-VUV radiation

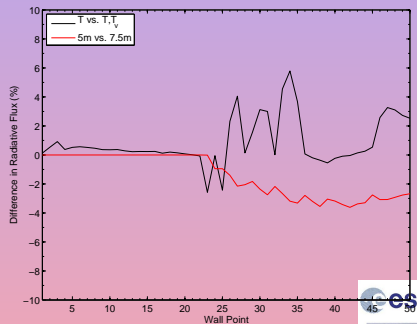
# Contribution of Radiative Fluxes above $1\mu\text{m}$ to the Overall Radiative Power; Large Spacecraft Configuration



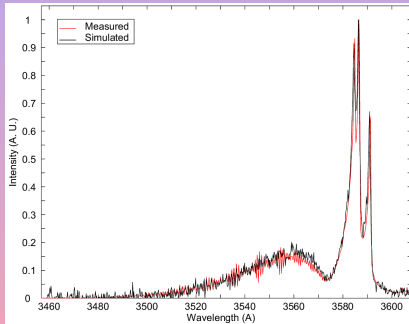
Values for the smaller spacecraft configuration are similar. Other studies in the scope of the ESA TC3 testcase confirm such findings for  $\text{CO}_2\text{-N}_2$  mixtures (CN and  $\text{C}_2$  radiation less than 3% overall in the stagnation streamline point).

# Sensitivity Studies

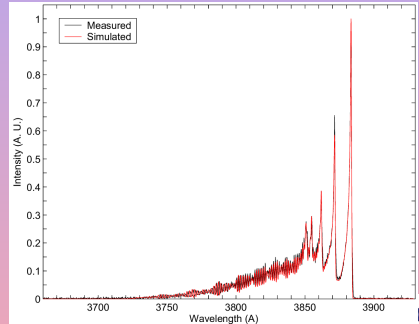
- Ray-tracing calculations carried for peak conductive heating (PH;  $T_v/T$  1.2) either considering (T,  $T_v$ ) or just (T).
- Ray-tracing calculations carried at 2.8km/s (P28) with a 5m and a 7.5m afterbody flow.
- Influence of thermal nonequilibrium less than 5% on wall fluxes, and enough backflow is accounted for (3% diff. max).



# A Careful Tailoring of Spectroscopic Databases Allows Very Good Fits of Equilibrium Radiation



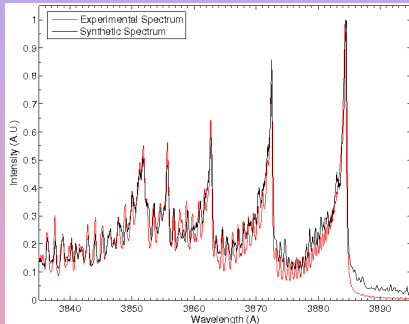
Best-fit to a measured CN Violet Spectrum ( $\Delta\nu = 1$ )



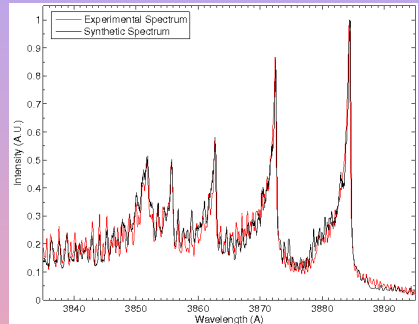
Best-fit to a measured CN Violet Spectrum ( $\Delta\nu = 0$ )

Spectra measured in equilibrium conditions in an ICP torch (97%  $\text{CO}_2$ –3%  $\text{N}_2$  gas)

# But Radiation is not Always Boltzmann!



Fitting of the CN Violet system, assuming  $T_r=10000K$  and a Boltzmann vibrational distribution

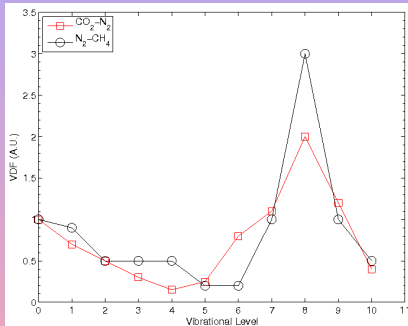


Fitting of the CN Violet system, assuming  $T_r=2700K$  and a non-Boltzmann vibrational distribution

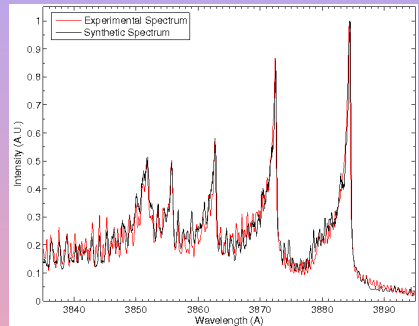
Fitting of a strongly nonequilibrium CN Violet Spectra obtained in a Supersonic Arc-Jet for a Titan-type (95%  $N_2$ -5%  $CH_4$  gas)



# But Radiation is not Always Boltzmann!



non-Boltzmann vibrational distribution Function



Fitting of the CN Violet system, assuming  $T_r=2700\text{K}$  and a non-Boltzmann vibrational distribution

Fitting of a strongly nonequilibrium CN Violet Spectra obtained in a Supersonic Arc-Jet for a Titan-type (95%  $\text{N}_2$ -5%  $\text{CH}_4$  gas)